# Circuit elements are classified into

Passive elements

1-Resistor

2- Inductor

3- Capacitor

Active elements such as

1-Voltage source like batteries

2-Current source

3-Generators

4-Operational amplifier

There are two types of elements found in electric circuits: passive elements and active elements. An active element is capable of generating energy while a passive element is not.

**1.1 Resistance of the material**:

The flow of charge through any material encounters an opposing force due to the collisions between electrons and between electrons and other atoms in the material, which converts electrical energy into another form of energy such as heat, is called the resistance of the material . The unit of measurement of resistance is the ohm, for which the symbol is (Ω). The circuit symbol for resistance appears in Fig. (1.1)

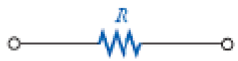



Figure 1.1

The resistance of any material with a uniform cross-sectional area is determined by the following four factors:

1. Material resistivity
2. Length
3. Cross-sectional area
4. Temperature

Conductors will have low resistance levels, while insulators will have high resistance characteristics. At a fixed temperature of 20°C (room temperature), the resistance is related to the other three factors by:

$$R = \rho \frac{l}{A} \quad \text{(ohms, } \Omega\text{)} \qquad (1\text{-}1)$$

where r (Greek letter rho) is a characteristic of the material called the resistivity, l is the length of the sample, and A is the cross-sectional area of the sample.

Table 1-1

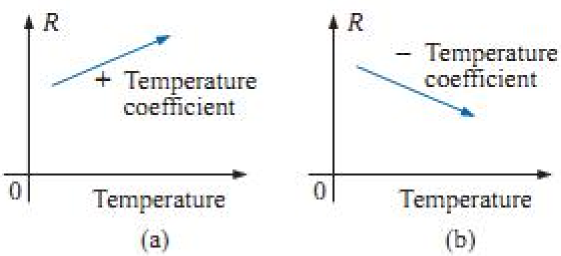
*Resistivity (ρ) of various materials.*

| Material | ρ @ 20°C |
|---|---|
| Silver | 9.9 |
| Copper | **10.37** |
| Gold | 14.7 |
| Aluminum | 17.0 |
| Tungsten | 33.0 |
| Nickel | 47.0 |
| Iron | 74.0 |
| Constantan | 295.0 |
| Nichrome | 600.0 |
| Calorite | 720.0 |
| Carbon | 21,000.0 |

## 1.2 TEMPERATURE EFFECTS

Temperature has a significant effect on the resistance of conductors, semiconductors, and insulators. Conductors have a generous number of free electrons, and any introduction of thermal energy will have little impact on the total number of free carriers. In fact, the thermal energy will only increase the intensity of the random motion of the particles within the material and make it increasingly difficult for a general drift of electrons in any one direction to be established. The result is that for good conductors, an increase in temperature will result in an increase in the resistance level. so, conductors have a positive temperature coefficient.

**The plot of Fig.(1.2) . (a) Positive temperature coefficient—conductors; (b) negative temperature coefficient—semiconductors.**




it is important that we have some method of determining the resistance at any temperature within operating limits. At two different temperatures, $T_1$ and $T_2$, the resistance of copper is

$R_1$ and $R_2$ , we may develop a mathematical relationship between these values of resistances at different temperatures. as shown in equation (1-2).

$$\frac{234.5 + T_1}{R_1} = \frac{234.5 + T_2}{R_2} \qquad (1\text{-}2)$$

$$\frac{|T_1| + T_1}{R_1} = \frac{|T_1| + T_2}{R_2} \qquad (1\text{-}3)$$

Table (1-2)

*Inferred absolute temperatures ($T_i$).*

| Material | °C |
|---|---|
| Silver | −243 |
| **Copper** | **−234.5** |
| Gold | −274 |
| Aluminum | −236 |
| Tungsten | −204 |
| Nickel | −147 |
| Iron | −162 |
| Nichrome | −2,250 |
| Constantan | −125,000 |

where $|T_1|$ indicates that the inferred absolute temperature of the material involved is inserted as a positive value in the equation. In general, therefore, associate the sign only with $T_1$ and $T_2$ .

EXAMPLE 1.1 If the resistance of a copper wire is 50 at 20°C, what is its resistance at 100°C (boiling point of water)?

Solution:

$$\frac{234.5°C + 20°C}{50\ \Omega} = \frac{234.5°C + 100°C}{R_2}$$

$$R_2 = \frac{(50\ \Omega)(334.5°C)}{254.5°C} = \mathbf{65.72\ \Omega}$$

EXAMPLE 1.2 If the resistance of a copper wire at freezing (0°C) is 30 ,what is its resistance at 40°C?

Solution:

$$\frac{234.5°C + 0}{30\ \Omega} = \frac{234.5°C - 40°C}{R_2}$$

$$R_2 = \frac{(30\ \Omega)(194.5°C)}{234.5°C} = \mathbf{24.88\ \Omega}$$

EXAMPLE 1.3 If the resistance of an aluminum wire at room temperature (20°C) is 100 m (measured by a milliohm meter), at what temperature will its resistance increase to 120 m?

Solution:

$$\frac{236°C + 20°C}{100\ m\Omega} = \frac{236°C + T_2}{120\ m\Omega}$$

and

$$T_2 = 120\ m\Omega\left(\frac{256°C}{100\ m\Omega}\right) - 236°C$$

$$T_2 = \mathbf{71.2°C}$$

## 1.3 Temperature Coefficient of Resistance

There is a second popular equation for calculating the resistance of a conductor at different temperatures. Defining :

$$\alpha_{20} = \frac{1}{|T_1| + 20°C} \quad (\Omega/°C/\Omega) \qquad (1\text{-}4)$$

as the temperature coefficient of resistance at a temperature of 20°C, and R20 as the resistance of the sample at 20°C, the resistance $R_1$ at a temperature $T_1$ is determined by

$$R_1 = R_{20}[1 + \alpha_{20}(T_1 - 20°C)] \qquad (1\text{-}5)$$

Equation (1-4) can be written in the following form:

$$\alpha_{20} = \frac{\left(\frac{R_1 - R_{20}}{T_1 - 20°C}\right)}{R_{20}} = \frac{\frac{\Delta R}{\Delta T}}{R_{20}} \qquad (1\text{-}6)$$

The values of a20 for different materials have been evaluated, and a few are listed in Table

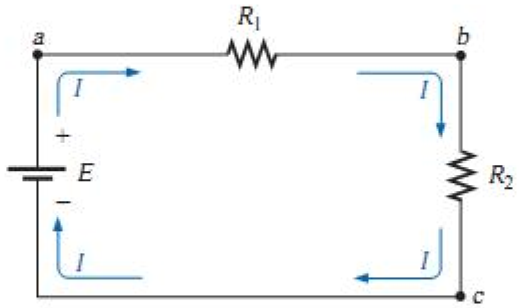
Table (1-3)

*Temperature coefficient of resistance for various conductors at 20°C.*

| Material | Temperature Coefficient ($\alpha_{20}$) |
|---|---|
| Silver | 0.0038 |
| Copper | 0.00393 |
| Gold | 0.0034 |
| Aluminum | 0.00391 |
| Tungsten | 0.005 |
| Nickel | 0.006 |
| Iron | 0.0055 |
| Constantan | 0.000008 |
| Nichrome | 0.00044 |

SERIES CIRCUITS:

A circuit consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. below, has three elements joined at three terminal points (a, b, and c) to provide a closed path for the current I.




-The current is the same through series elements.

-The current is the same through series elements.

- to find the total resistance of N resistors in series, the following equation is applied:

$$R_T = R_1 + R_2 + R_3 + \cdots + R_N \qquad \text{(ohms, } \Omega\text{)}$$

To find the total resistance of N resistors of the same value in series, simply multiply the value of one of the resistors by the number in series; that is,

$$R_T = NR$$

the current drawn from the source can be determined using Ohm's law, as follows:

$$I_s = \frac{E}{R_T} \qquad \text{(amperes, A)}$$

the voltage across each resistor with the total resistance. using Ohm's law; that is,

$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3,\ \ldots,\ V_N = IR_N \qquad \text{(volts, V)}$$

The power delivered to each resistor can then be determined using any one of three equations as listed below for R1:

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \qquad \text{(watts, W)}$$

The power delivered by the source is:

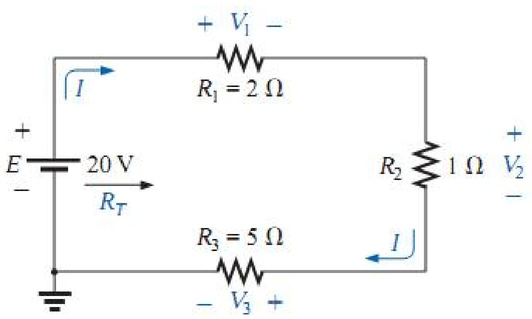
$$P_{del} = EI \qquad \text{(watts, W)}$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

That is,

$$P_{del} = P_1 + P_2 + P_3 + \cdots + P_N$$

EXAMPLE: for Figure




a) find the total resistance for the series circuit of
b) Calculate the source current I .
c) Determine the voltages $V_1$ , $V_2$ , and $V_3$ .
d) Calculate the power dissipated by $R_1$ , $R_2$ , and $R_3$ .
e) Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

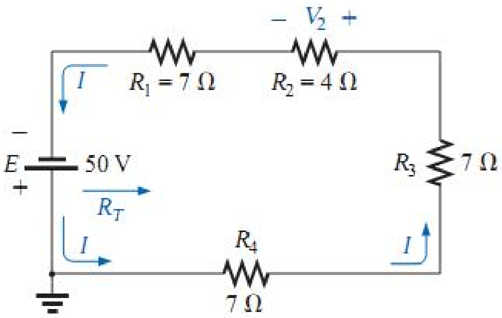
b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ \text{V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\ \text{A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\ \text{A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\ \text{A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1 I_1 = (5\ \text{V})(2.5\ \text{A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2 R_2 = (2.5\ \text{A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\ \text{V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{\text{del}} = EI = (20\ \text{V})(2.5\ \text{A}) = \mathbf{50\ W}$
$P_{\text{del}} = P_1 + P_2 + P_3$
$50\ \text{W} = 12.5\ \text{W} + 6.25\ \text{W} + 31.25\ \text{W}$
$\underline{50\ \text{W} = 50\ \text{W} \quad \text{(checks)}}$

EXAMPLE : Determine $R_T$ , I, and $V_2$ for the circuit of Fig. below




Solution: Note the current direction as established by the battery and the polarity of the voltage drops across $R_2$ as determined by the current direction. Since
$R_1 = R_3 = R_4$ ,

$$R_T = NR_1 + R_2 = (3)(7\ \Omega) + 4\ \Omega = 21\ \Omega + 4\ \Omega = \mathbf{25\ \Omega}$$

$$I = \frac{E}{R_T} = \frac{50\ \text{V}}{25\ \Omega} = \mathbf{2\ A}$$

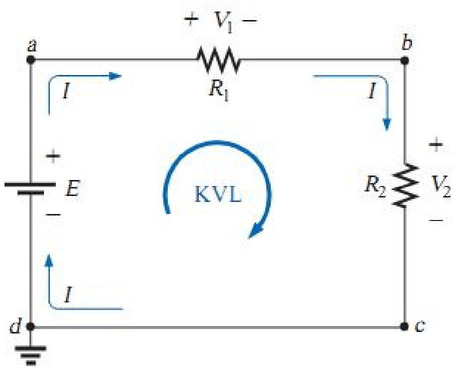
$$V_2 = IR_2 = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$

KIRCHHOFF’S VOLTAGE LAW:

<u>Kirchhoff’s voltage law (KVL)</u> :states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

<u>A closed loop</u>: is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

In Fig. 5.12, by following the current, we can trace a continuous path that leaves point a through $R_1$ and returns through E without leaving the circuit. Therefore, abcda is a closed loop. For us to be able to apply Kirchhoff’s voltage law, the summation of potential rises and drops must be made in one direction around the closed loop.




For uniformity, the clockwise (CW) direction will be used throughout the text for all applications of Kirchhoff’s voltage law.

$$\Sigma_{\circlearrowright} V = 0$$ (Kirchhoff’s voltage law in symbolic form)

which for the circuit of Fig. yields (clockwise direction, following the current I and starting at point d):

$$+E - V_1 - V_2 = 0$$

$$E = V_1 + V_2$$

the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

Kirchhoff's voltage law can also be stated in the following form:

$$\Sigma_{\circlearrowright} V_{rises} = \Sigma_{\circlearrowright} V_{drops}$$

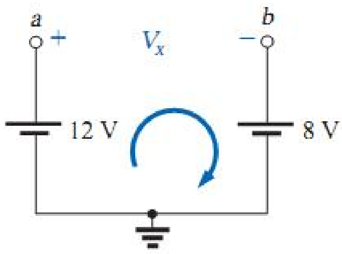
If the loop were taken in the counterclockwise direction starting at point a, the following would result:

$$\Sigma_{\circlearrowright} V = 0$$
$$-E + V_2 + V_1 = 0$$
$$E = V_1 + V_2$$

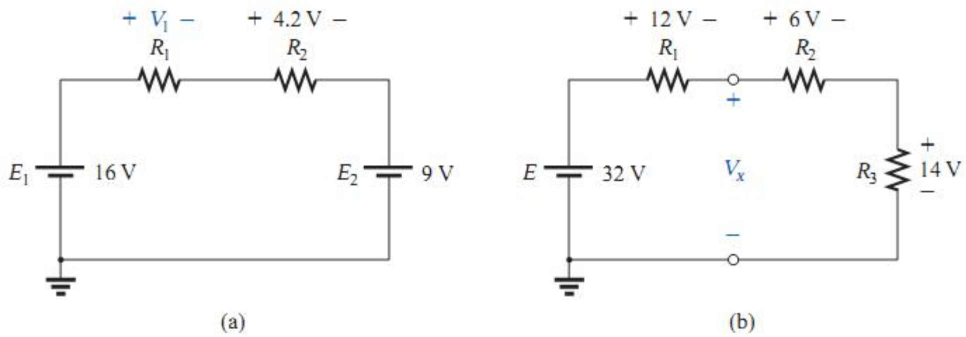
The application of Kirchhoff's voltage law need not follow a path that includes current-carrying elements.

For example, in Fig. 5.13 there is a difference in potential between points a and b, even though the two points are not connected by a current-carrying element. Application of Kirchhoff's voltage law around the closed loop will result in a difference in potential of 4 V between the two points. That is, using the clockwise direction:

$$+12\text{ V} - V_x - 8\text{ V} = 0$$
$$V_x = \mathbf{4\ V}$$




EXAMPLE 5.4 Determine the unknown voltages for the networks of Fig.




Solution (a):

$$+E_1 - V_1 - V_2 - E_2 = 0$$

$$V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$
$$= \mathbf{2.8\ V}$$

Solution(b):

$$+E - V_1 - V_x = 0$$

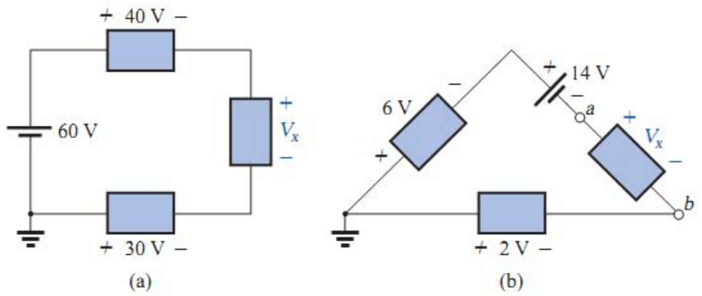
$$V_x = E - V_1 = 32\text{ V} - 12\text{ V}$$
$$= \mathbf{20\ V}$$

Or

$$+V_x - V_2 - V_3 = 0$$

$$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V}$$
$$= \mathbf{20\ V}$$

EXAMPLE : Using Kirchhoff's voltage law, determine the unknown voltages for the network shown below.



Solution (a):

$$60\text{ V} - 40\text{ V} - V_x + 30\text{ V} = 0$$

$$V_x = 60\text{ V} + 30\text{ V} - 40\text{ V} = 90\text{ V} - 40\text{ V}$$
$$= \mathbf{50\ V}$$

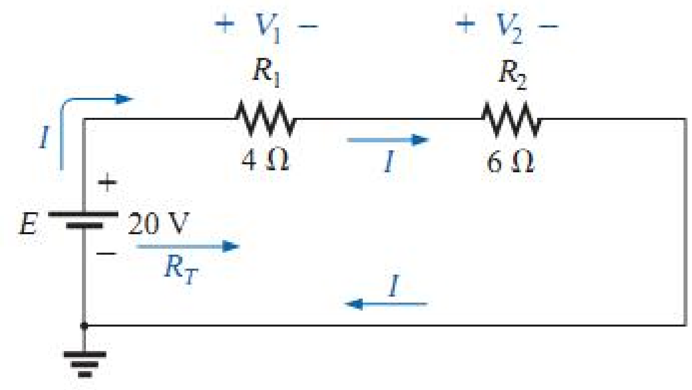
Solution (b):

$$-6\text{ V} - 14\text{ V} - V_x + 2\text{ V} = 0$$

$$V_x = -20\text{ V} + 2\text{ V}$$
$$= \mathbf{-18\ V}$$

Since the result is negative, we know that a should be negative and b should be positive, but the magnitude of 18 V is correct.

EXAMPLE : For the circuit shown




a) Find $R_T$ .
b) Find I.
c) Find $V_1$ and $V_2$.
d) Find the power to the 4Ω and 6Ω resistors.
e) Find the power delivered by the battery, and compare it to that dissipated by the 4Ω and 6Ω resistors combined.
f) Verify Kirchhoff's voltage law (clockwise direction).

Solution:

a. $R_T = R_1 + R_2 = 4\ \Omega + 6\ \Omega = \mathbf{10\ \Omega}$

b. $I = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{10\ \Omega} = \mathbf{2\ A}$

c. $V_1 = IR_1 = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$
$V_2 = IR_2 = (2\text{ A})(6\ \Omega) = \mathbf{12\ V}$

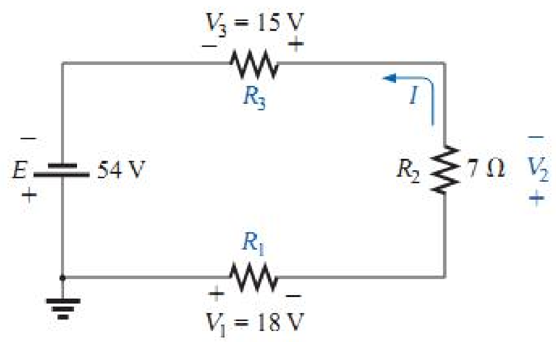
d. $P_{4\Omega} = \dfrac{V_1^2}{R_1} = \dfrac{(8\text{ V})^2}{4} = \dfrac{64}{4} = \mathbf{16\ W}$

$P_{6\Omega} = I^2R_2 = (2\text{ A})^2(6\ \Omega) = (4)(6) = \mathbf{24\ W}$

e. $P_E = EI = (20\text{ V})(2\text{ A}) = \mathbf{40\ W}$
$P_E = P_{4\Omega} + P_{6\Omega}$
$40\text{ W} = 16\text{ W} + 24\text{ W}$
$40\text{ W} = 40\text{ W}$ (checks)

f. $\Sigma_{C} V = +E - V_1 - V_2 = 0$
$E = V_1 + V_2$
$20\text{ V} = 8\text{ V} + 12\text{ V}$
$20\text{ V} = 20\text{ V}$ (checks)

EXAMPLE : For the circuit of Fig. shown:




a) Determine $V_2$ using Kirchhoff's voltage law.
b) Determine I.
c) Find $R_1$ and $R_3$.

Solutions:

a) Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or $$E = V_1 + V_2 + V_3$$

and $V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\text{ V}}$

b. $I = \dfrac{V_2}{R_2} = \dfrac{21\text{ V}}{7\ \Omega} = \mathbf{3\text{ A}}$

c. $R_1 = \dfrac{V_1}{I} = \dfrac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

$R_3 = \dfrac{V_3}{I} = \dfrac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

voltage divider rule : a method referred to as the voltage divider rule (VDR) that permits determining the voltage levels without first finding the current. The rule can be derived by analyzing the network shown.

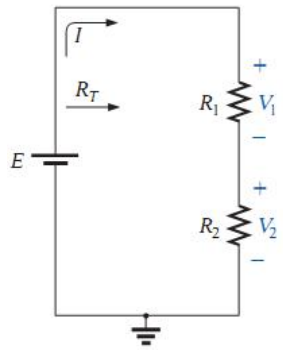
$$R_T = R_1 + R_2$$

and $$I = \frac{E}{R_T}$$

Applying Ohm's law:

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right)R_1 = \frac{R_1 E}{R_T}$$

with $$V_2 = IR_2 = \left(\frac{E}{R_T}\right)R_2 = \frac{R_2 E}{R_T}$$



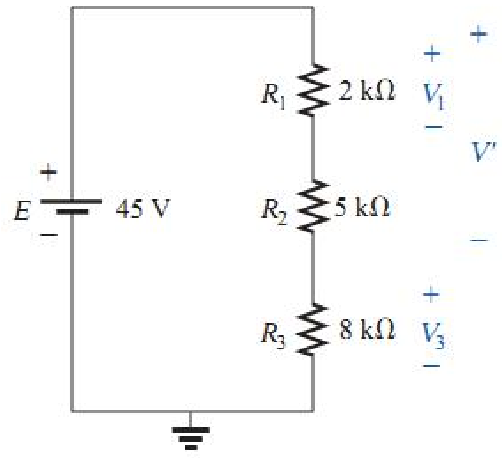

Note that the format for $V_1$ and $V_2$ is

$$V_x = \frac{R_x E}{R_T} \quad \text{(voltage divider rule)}$$

where $V_x$ is the voltage across $R_x$ , E is the impressed voltage across the series elements, and RT is the total resistance of the series circuit

The voltage divider rule (VDR) :states that the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

EXAMPLE 5.11 Using the voltage divider rule, determine the voltages $V_1$ and V3 for the series circuit shown in Fig.




Solution:

$$V_1 = \frac{R_1 E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \mathbf{6\ V}$$

$$V_3 = \frac{R_3 E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

$$= \frac{360\ \text{V}}{15} = \mathbf{24\ V}$$

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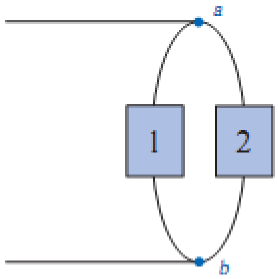
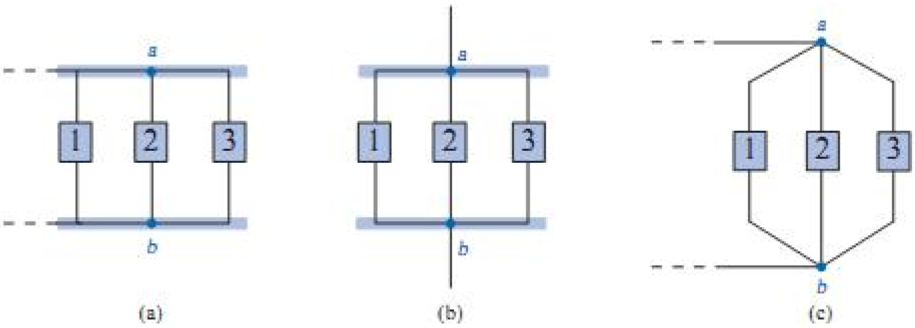
## Parallel Circuits

Two elements, branches, or networks are inparallel if they have two

points in common. In Fig. shown below, for example, elements 1 and 2 have terminals a and b in common; they are therefore in parallel.

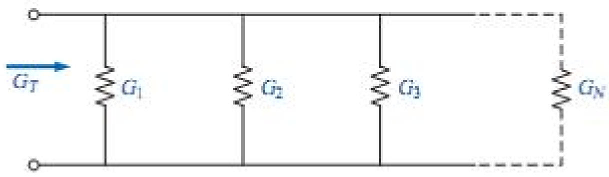



Different ways in which three parallel elements may appear in this Figures.



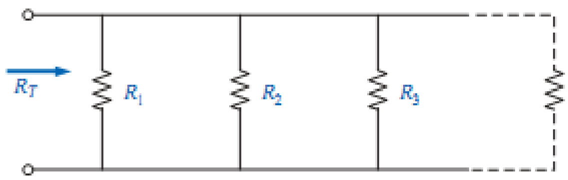

**-Total Conductance and Resistance**

For parallel elements, the total conductance is the sum of the individual conductance. That is, for the parallel network of Fig. , we write:




$$G_T = G_1 + G_2 + G_3 + \cdots + G_N$$

Since G =1/R, the total resistance for the network can be determined by:



Determining the total resistance of parallel resistors.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

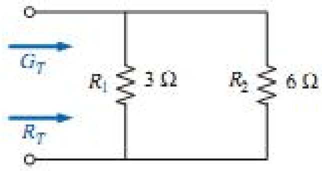
**EXAMPLE** : Determine the total conductance and resistance for the parallel network of Fig. shown.

Solution:

$$G_T = G_1 + G_2 = \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega} = 0.333\text{ S} + 0.167\text{ S} = \mathbf{0.5\text{ S}}$$

and

$$R_T = \frac{1}{G_T} = \frac{1}{0.5\text{ S}} = \mathbf{2\,\Omega}$$




**EXAMPLE** : Determine the effect on the total conductance and resistance of the network of previous Fig. if another resistor of 10 Ω were added in parallel with the other elements.

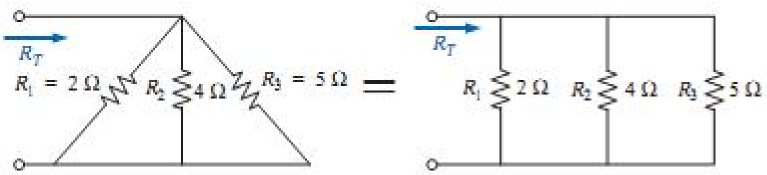
Solution:

$$G_T = 0.5\text{ S} + \frac{1}{10\,\Omega} = 0.5\text{ S} + 0.1\text{ S} = \mathbf{0.6\text{ S}}$$

$$R_T = \frac{1}{G_T} = \frac{1}{0.6\text{ S}} \cong \mathbf{1.667\,\Omega}$$

*Note, as mentioned above, that adding additional terms increases the conductance level and decreases the resistance level.

**EXAMPLE** : Determine the total resistance for the network of Fig. below.




Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{5\ \Omega} = 0.5\ \text{S} + 0.25\ \text{S} + 0.2\ \text{S}$$

$$= 0.95\ \text{S}$$

and $$R_T = \frac{1}{0.95\ \text{S}} = \mathbf{1.053\ \Omega}$$

The above examples demonstrate an interesting and useful (for checking purposes) characteristic of parallel resistors:

*The total resistance of parallel resistors is always less than the value of the smallest resistor.*

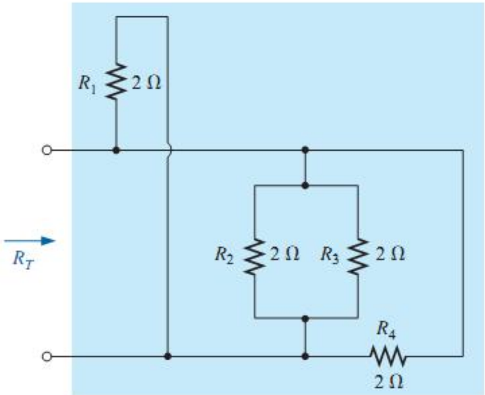
-For equal resistors in parallel, the equation for the total resistance becomes significantly easier to apply. For N equal resistors in parallel :

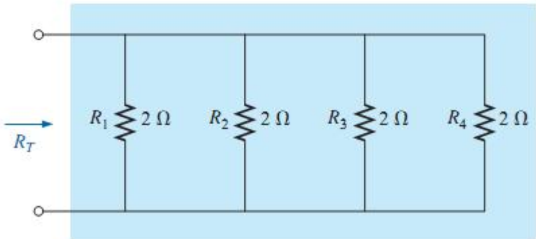
$$R_T = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R_N}}$$

$$= \frac{1}{N\left(\frac{1}{R}\right)} = \frac{1}{\frac{N}{R}}$$

$$\boxed{R_T = \frac{R}{N}}$$

**EXAMPLE** : Find the total resistance for the configuration in Fig.



Solution: Redrawing the network

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

Special Case: Two Parallel Resistors

For two parallel resistors, the total resistance is determined by Eq. :

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying the top and bottom of each term of the right side of the equation by the other resistor results in

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}$$

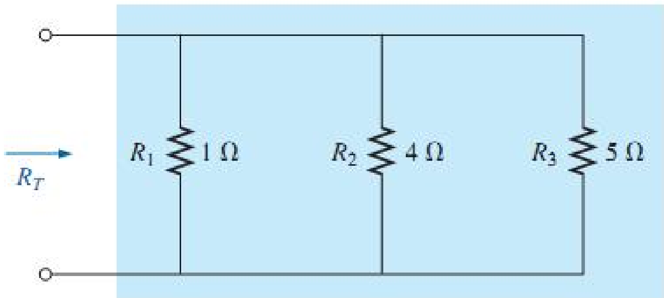
$$\frac{1}{R_T} = \frac{R_2 + R_1}{R_1R_2}$$

$$R_T = \frac{R_1R_2}{R_1 + R_2}$$

In words, the equation states that

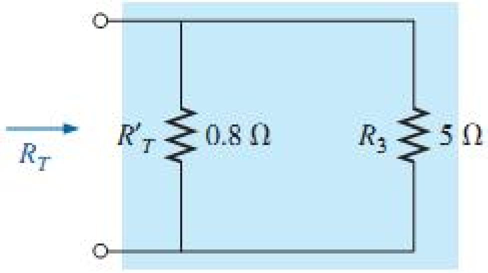
the total resistance of two parallel resistors is simply the product of their values divided by their sum.

EXAMPLE : Determine the total resistance for the parallel combination in Fig.




Solution:

First the 1Ω and 4Ω resistors are combined, resulting in the reduced network in Fig.




$$R'_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(1\ \Omega)(4\ \Omega)}{1\ \Omega + 4\ \Omega} = \frac{4}{5}\ \Omega = 0.8\ \Omega$$

Then,

$$R_T = \frac{R'_T R_3}{R'_T + R_3} = \frac{(0.8\ \Omega)(5\ \Omega)}{0.8\ \Omega + 5\ \Omega} = \frac{4}{5.8}\ \Omega = \mathbf{0.69\ \Omega}$$

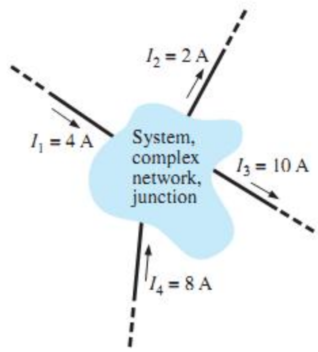
**KIRCHHOFF'S CURRENT LAW (KCL):**

The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

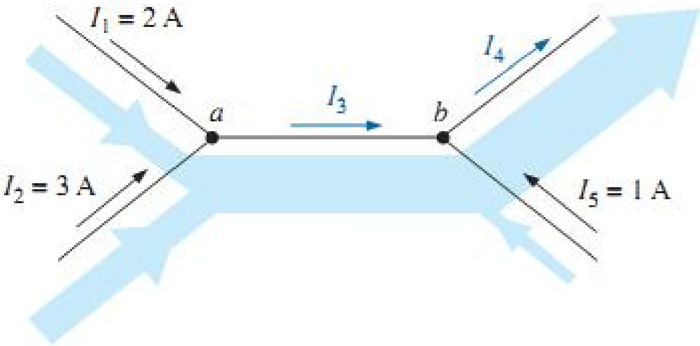
$$\Sigma I_i = \Sigma I_o$$

In Fig. below, for example, the shaded area can enclose an entire system or a complex network, or it can simply provide a connection point (junction) for the displayed currents. In each case, the current entering must equal that leaving,

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$\mathbf{12\text{ A} = 12\text{ A}} \quad \text{(checks)}$$



**EXAMPLE** : Determine currents $I_3$ and $I_4$ in Fig. using Kirchhoff's current law.




Solution:

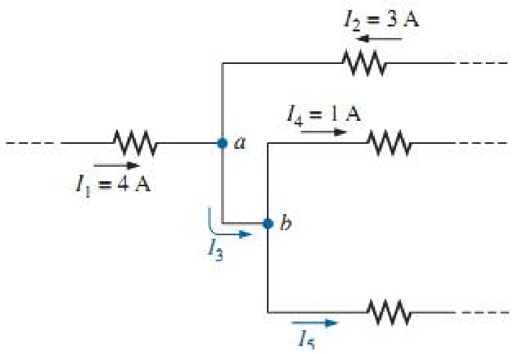
At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3 = \mathbf{5\ A}$$

At node *b*, using the result just obtained:

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4 = \mathbf{6\ A}$$

EXAMPLE : Determine currents I3 and I5 in Fig. through applications of Kirchhoff's current law.




Solution:

At node *a:*

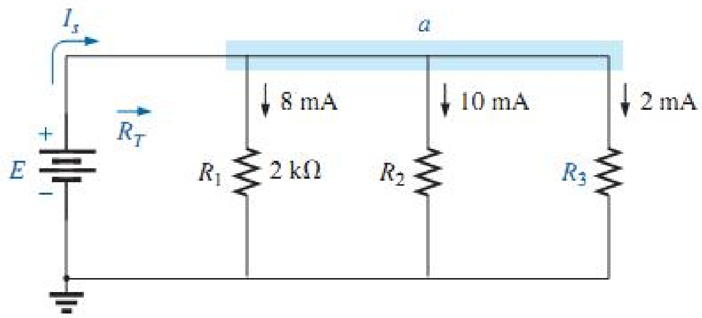
$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$4\text{ A} + 3\text{ A} = I_3 = \mathbf{7\text{ A}}$$

At node *b:*

$$\Sigma I_i = \Sigma I_o$$
$$I_3 = I_4 + I_5$$
$$7\text{ A} = 1\text{ A} + I_5$$

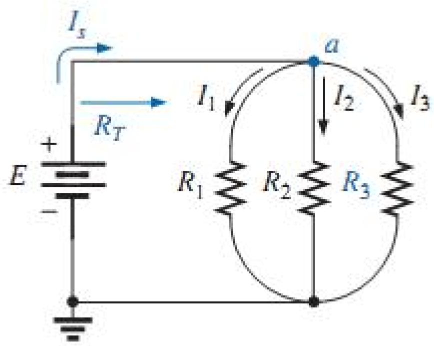
and $$I_5 = 7\text{ A} - 1\text{ A} = \mathbf{6\text{ A}}$$

EXAMPLE : For the parallel dc network in Fig.




a. Determine the source current $I_s$ .
b. Find the source voltage E.
c. Determine $R_3$.
d. Calculate $R_T$

Solution:



a)

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 8\text{ mA} + 10\text{ mA} + 2\text{ mA} = \mathbf{20\text{ mA}}$$

b)

$$E = V_1 = I_1 R_1 = (8\text{ mA})(2\text{ k}\Omega) = \mathbf{16\text{ V}}$$

c)

$$R_3 = \frac{V_3}{I_3} = \frac{E}{I_3} = \frac{16\text{ V}}{2\text{ mA}} = \mathbf{8\text{ k}\Omega}$$

d)

$$R_T = \frac{E}{I_s} = \frac{16\text{ V}}{20\text{ mA}} = \mathbf{0.8\text{ k}\Omega}$$

**CURRENT DIVIDER RULE:**

We now introduce the equally powerful current divider rule (CDR) for finding the current through a resistor in a parallel circuit.

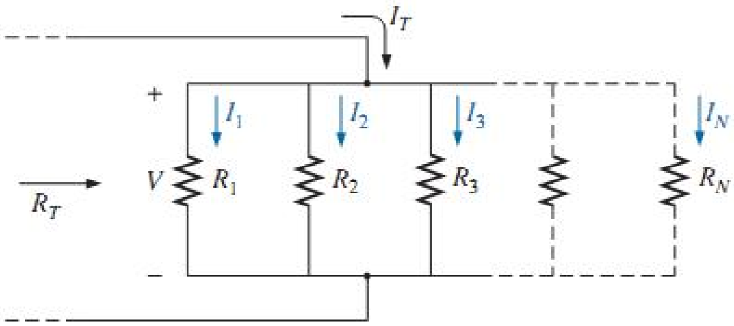
-For two parallel elements of equal value, the current will divide equally.

-For parallel elements with different values, the smaller the resistance, the greater the share of input current.

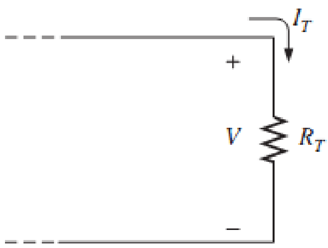
-For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

Deriving the current divider rule:

a) parallel network of N parallel resistors




b) reduced equivalent of Fig. in (a).




The current $I_T$ can then be determined using Ohm's law:

$$I_T = \frac{V}{R_T}$$

Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

where the product $I_x$ $R_x$ ,refers to any combination in the series.

Substituting for V in the above equation for $I_T$, we have

$$I_T = \frac{I_xR_x}{R_T}$$

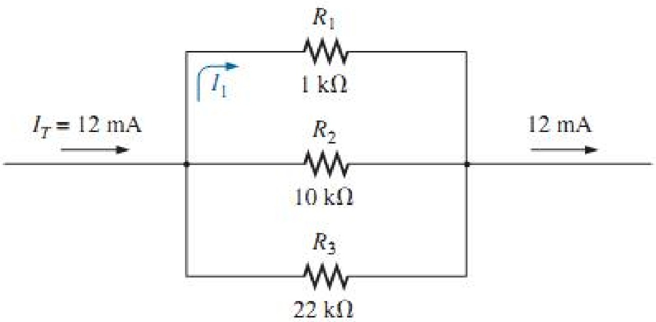
Solving for $I_x$ , the final result is the **current divider rule**:

$$I_x = \frac{R_T}{R_x}I_T$$

which states that:

The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.

EXAMPLE : For the parallel network in Fig., determine current $I_1$ using (CDR).




Solution:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{1\ \text{k}\Omega} + \frac{1}{10\ \text{k}\Omega} + \frac{1}{22\ \text{k}\Omega}}$$

$$= \frac{1}{1 \times 10^{-3} + 100 \times 10^{-6} + 45.46 \times 10^{-6}}$$

$$= \frac{1}{1.145 \times 10^{-3}} = \mathbf{873.01\ \Omega}$$

and the smallest parallel resistor receives the majority of the current.

$$I_1 = \frac{R_T}{R_1} I_T$$

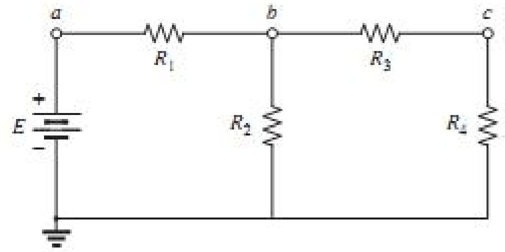
$$= \frac{(873.01\ \Omega)}{1\ \text{k}\Omega}(12\ \text{mA}) = (0.873)(12\ \text{mA}) = \mathbf{10.48\ mA}$$

**SERIES-PARALLEL NETWORKS:**

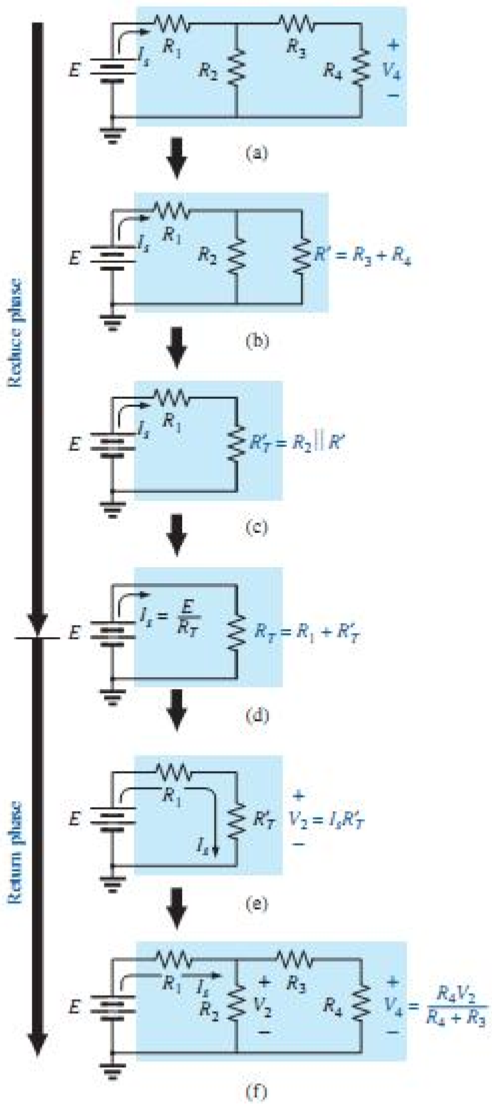
A series-parallel configuration is one that is formed by a combination of series and parallel elements.

A complex configuration is one in which none of the elements are in series or parallel.

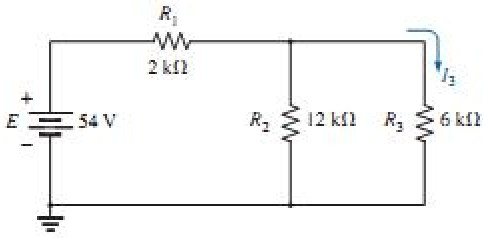
The network in Fig. shown is a series-parallel network. At first, you must be very careful to determine which elements are in series and which are in parallel.




series-parallel network

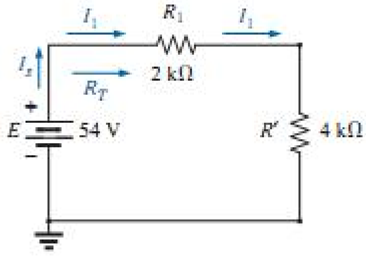


Example : Find current $I_3$ for the series-parallel network in Fig. below.




Solution:

$$R' = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{(12\text{ k}\Omega)(6\text{ k}\Omega)}{12\text{ k}\Omega + 6\text{ k}\Omega} = 4\text{ k}\Omega$$




Substituting the parallel equivalent resistance for

resistors $R_2$ and $R_3$

$$R_T = R_1 + R' = 2\text{ k}\Omega + 4\text{ k}\Omega = 6\text{ k}\Omega$$

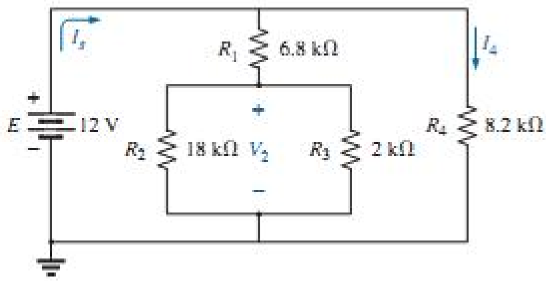
The source current is then determined using Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{54\text{ V}}{6\text{ k}\Omega} = 9\text{ mA}$$

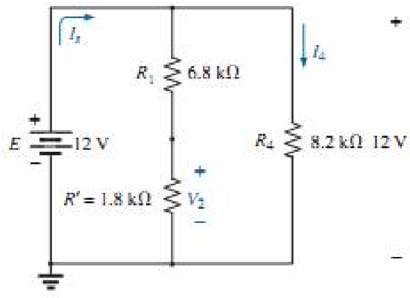
I1 is the total current entering the parallel combination of $R_2$ and $R_3$ . Applying the current divider rule results in the desired current:

$$I_3 = \left(\frac{R_2}{R_2 + R_3}\right)I_1 = \left(\frac{12\text{ k}\Omega}{12\text{ k}\Omega + 6\text{ k}\Omega}\right)9\text{ mA} = \mathbf{6\text{ mA}}$$

Example : For the network in Fig. below. Determine currents $I_4$ and $I_s$ and voltage $V_2$.



Solution:




$$R' = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{(18\ \text{k}\Omega)(2\ \text{k}\Omega)}{18\ \text{k}\Omega + 2\ \text{k}\Omega} = 1.8\ \text{k}\Omega$$

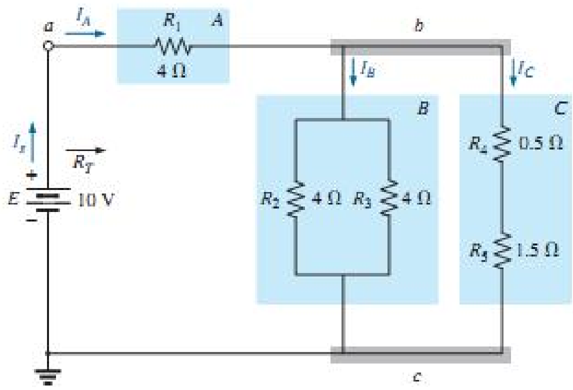
$$I_4 = \frac{V_4}{R_4} = \frac{E}{R_4} = \frac{12\ \text{V}}{8.2\ \text{k}\Omega} = \mathbf{1.46\ mA}$$

$$V_2 = \left(\frac{R'}{R' + R_1}\right)E = \left(\frac{1.8\ \text{k}\Omega}{1.8\ \text{k}\Omega + 6.8\ \text{k}\Omega}\right)12\ \text{V} = \mathbf{2.51\ V}$$

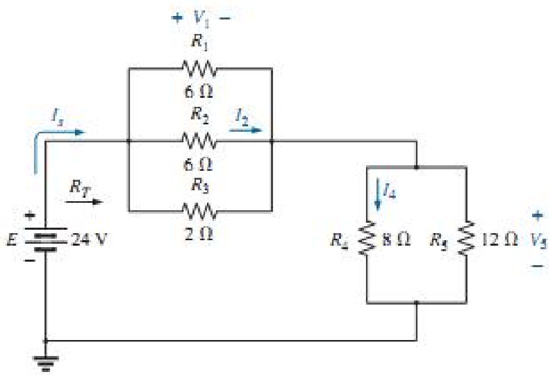
$$I_1 = \frac{E}{R_1 + R'} = \frac{12\ \text{V}}{6.8\ \text{k}\Omega + 1.8\ \text{k}\Omega} = 1.40\ \text{mA}$$

and $I_s = I_1 + I_4 = 1.40\ \text{mA} + 1.46\ \text{mA} = \mathbf{2.86\ mA}$

H.W// Determine all the currents and voltages of the network in Fig. below.

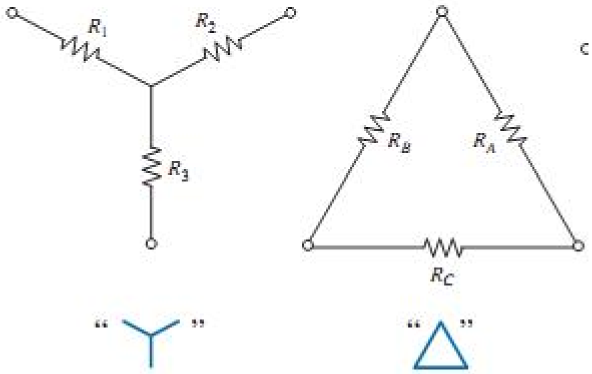



H.W// Find the indicated currents and voltages for the network in Fig.



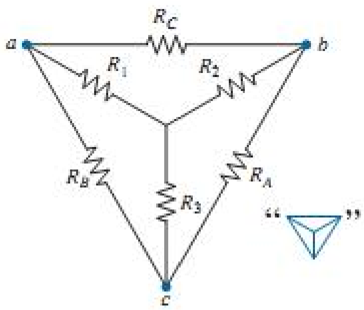
## Y- Δ AND Δ-Y CONVERSIONS:

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the wye (Y) and delta (Δ) configurations depicted in Fig.




The purpose of this section is to develop the equations for converting from Δ to Y, or vice versa.

**<u>Delta to Star :</u>**



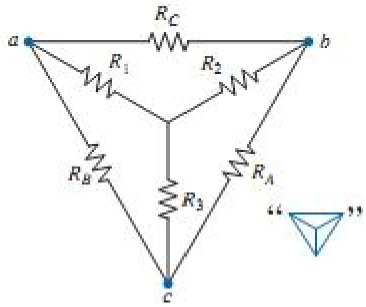

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

-***Note*** that each resistor of the Y is equal to the product of the resistors in the two closest branches of the  divided by the sum of the resistors in the Δ.

## Star to Delta:




$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

-***Note*** that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

Let us consider what would occur if all the values of a Δ or Y were the same. If $R_A = R_B = R_C$,

$$R_3 = \frac{R_AR_B}{R_A + R_B + R_C} = \frac{R_AR_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

and, following the same procedure,

$$R_1 = \frac{R_A}{3} \qquad R_2 = \frac{R_A}{3}$$

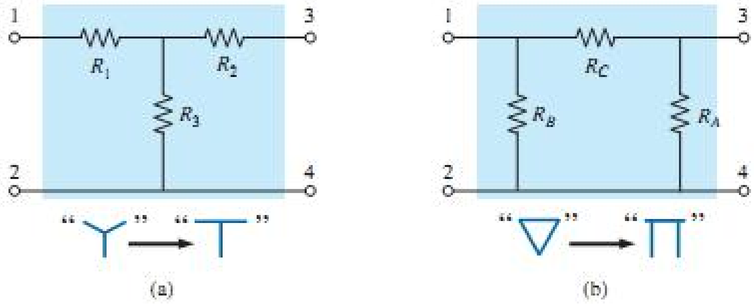
In general, therefore,

$$R_Y = \frac{R_\Delta}{3}$$

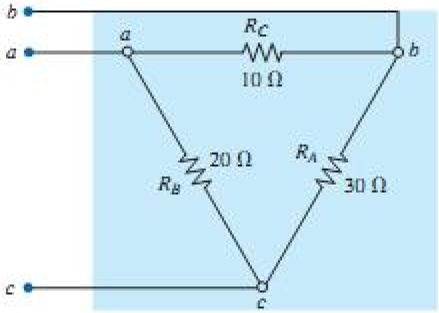
or

$$R_\Delta = 3R_Y$$

The Y and Δ the often appear. They are then referred to as a tee (T) and a pi (p) network, respectively. The equations used to convert from one form to the other are exactly the same as those developed for the Y and Δ transformation.




Example: Convert the Δ in Fig. to a Y.



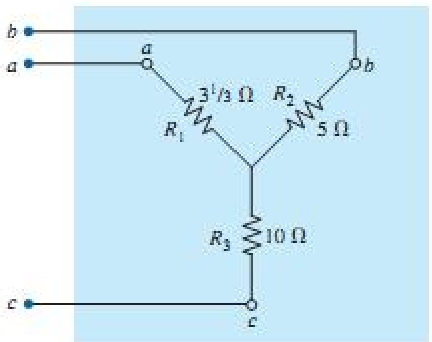

Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

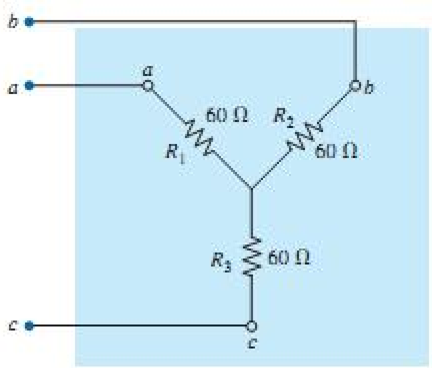
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

The equivalent network is shown in Fig. below.



Example : Convert the Y in Fig. to a Δ.




Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$
$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$
$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$
$$R_A = \mathbf{180\ \Omega}$$

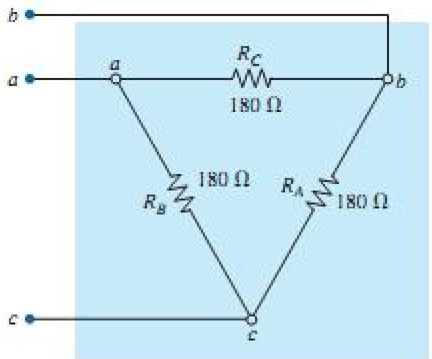
However, the three resistors for the Y are equal,

$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

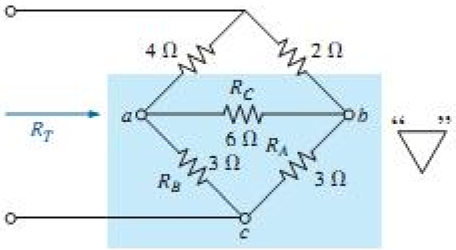
And

$$R_B = R_C = \mathbf{180\ \Omega}$$

The equivalent network is shown in Fig.



Example : Find the total resistance of the network in Fig. where $R_A = 3\Omega$ , $R_B = 3\Omega$ , and $R_C = 6\Omega$ .




Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

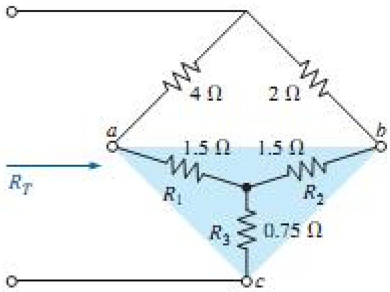
Replacing the Δ by the Y, as shown in Fig., yields:

$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

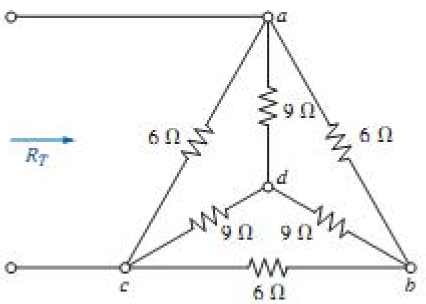
$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.89\ \Omega}$$

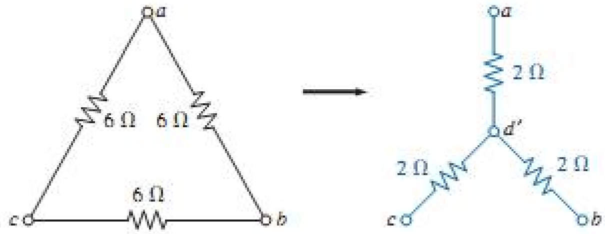



Example : Find the total resistance of the network in Fig.



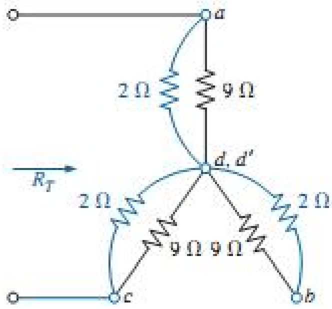
Solutions: Since all the resistors of the Δ or Y are the same,

a. Converting the Δ to a Y:




$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega$$

The network then appears as shown in Fig.




$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.27\ \Omega}$$

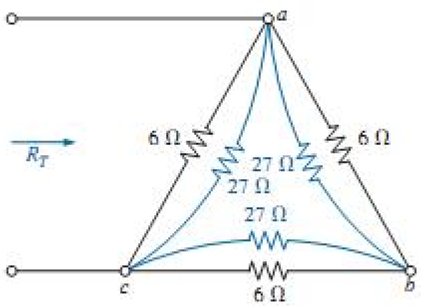
b. Converting the Y to a Δ :

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega$$

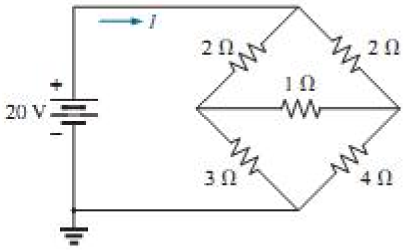
$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.91\ \Omega$$

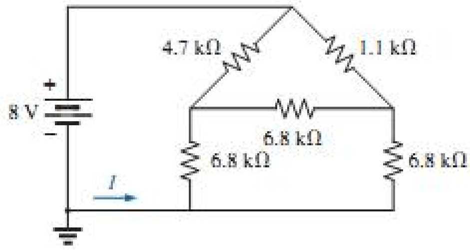
$$R_T = \frac{R'_T(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

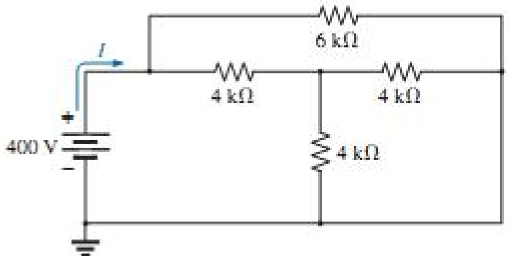
$$= \frac{2(4.91\ \Omega)}{3} = \mathbf{3.27\ \Omega}$$

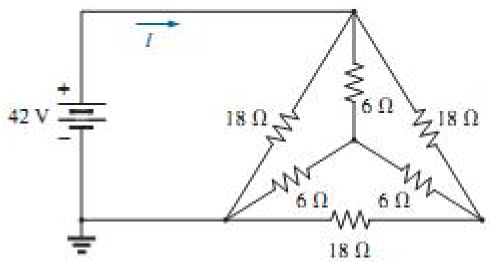


H.W// Using a Δ-Y or Y- Δ conversion, find the current I in each of  the networks



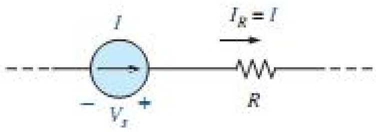



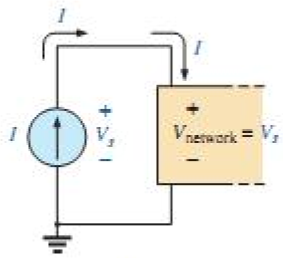



**CURRENT SOURCES:**

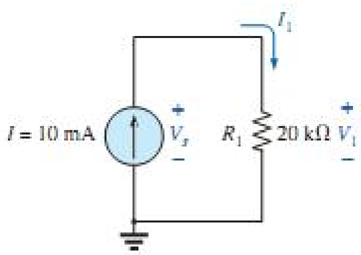
a current source determines the direction and magnitude of the current in the branch where it is located. the magnitude and the polarity of the voltage across a current source are each a function of the network to which the voltage is applied. A few examples will demonstrate the similarities between solving for the source current of a voltage source and the terminal voltage of a current source, so we just have to remember what we are looking for and properly understand the characteristics of each source. The symbol for a current source appears in Fig. (a). The arrow indicates the direction in which it is supplying current to the branch where it is located. The result is a current equal to the source current through the series resistor. In Fig. (b), we find that the voltage across a current source is determined by the polarity of the voltage drop caused by the current source. For single-source networks, it always has the polarity of Fig. (b).



(a)


(b)

EXAMPLE : Find the source voltage, the voltage $V_1$ , and current $I_1$ for the circuit in Fig.



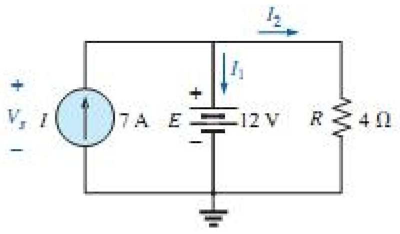

Solution:

$$I_1 = I = \mathbf{10\ mA}$$

$$V_1 = I_1R_1 = (10\ \text{mA})(20\ \Omega) = \mathbf{200\ V}$$

$$V_s = V_1 = \mathbf{200\ V}$$

EXAMPLE : Find the voltage $V_S$ and currents $I_1$ and $I_2$ for the network in Fig.




Solution:

Since the current source and voltage source are in parallel,

$$V_s = E = \mathbf{12\ V}$$

Further, since the voltage source and resistor $R$ are in parallel,

$$V_R = E = 12\ \text{V}$$

and $$I_2 = \frac{V_R}{R} = \frac{12\ \text{V}}{4\ \Omega} = \mathbf{3\ A}$$

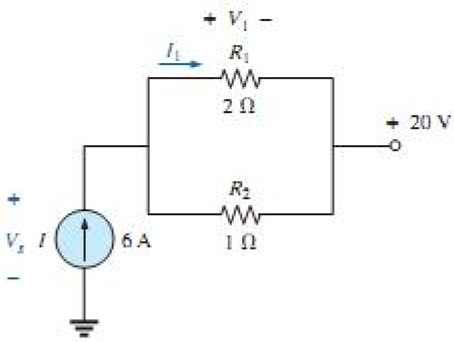
The current $I_1$ of the voltage source can then be determined by applying Kirchhoff's current law at the top of the network as follows:

$$\Sigma I_i = \Sigma I_o$$

$$I = I_1 + I_2$$

and $$I_1 = I - I_2 = 7\ \text{A} - 3\ \text{A} = \mathbf{4\ A}$$

EXAMPLE : Determine the current I1 and the voltage VS for the network in Fig.




Solution:

First note that the current in the branch with the current source must be 6 A, no matter what the magnitude of the voltage source to the right. In other words, the currents of the network are defined by I, $R_1$, and $R_2$ . However, the voltage across the current source is directly affected by the magnitude and polarity of the applied source.

Using the current divider rule:

$$I_1 = \frac{R_2 I}{R_2 + R_1} = \frac{(1\ \Omega)(6\ \text{A})}{1\ \Omega + 2\ \Omega} = \frac{1}{3}(6\ \text{A}) = \mathbf{2\ A}$$

The voltage $V_1$:

$$V_1 = I_1 R_1 = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

Applying Kirchhoff's voltage rule to determine $V_s$:

$$+V_s - V_1 - 20\ \text{V} = 0$$

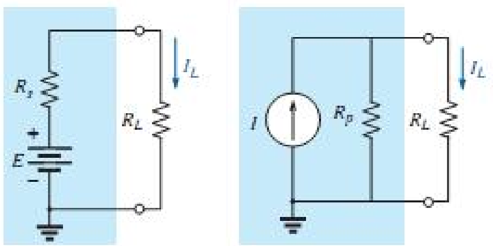
and

$$V_s = V_1 + 20\ \text{V} = 4\ \text{V} + 20\ \text{V} = \mathbf{24\ V}$$

In particular, note the polarity of the voltage $V_s$ as determined by the network.

**SOURCE CONVERSIONS:**

The current source appearing is the previous section is called an source due to the absence of any internal resistance. In reality, all sources whether they are voltage sources or current sources have internal resistance in the relative positions shown in Fig. For the voltage source, if Rs = 0 , or if it is so small compared to any series resistors that it can be ignored, then we have an "ideal" voltage source for all practical purposes. For the current source, since the resistor RP is in parallel, if RP = ∞ , or if it is large enough compared to any parallel resistive elements that it can be ignored, then we have an "ideal" current source. Unfortunately, however, ideal sources cannot be converted from one type to another. That is, a voltage source cannot be converted to a current source, and vice versa—the internal resistance must be present. If the voltage source in Fig.(a) is to be equivalent to the source in Fig. (b).

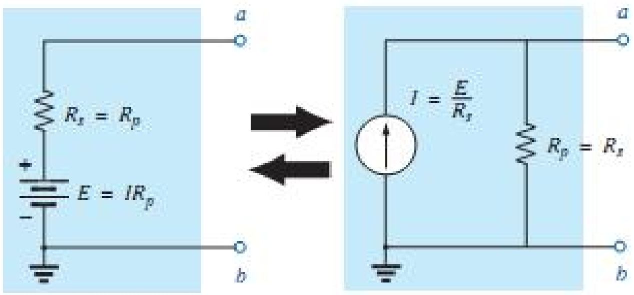



(a) (b)

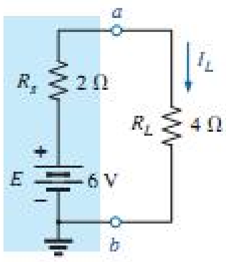
the load $R_L$ would not know which source it was connected to. This type of equivalence is established using the equations appearing in Fig. below. First note that the resistance is the same in each configuration a nice advantage. For the voltage source equivalent, the voltage is determined by a simple application of Ohm's law to the current source $E = IR_P$. For the current source equivalent, the current is again deter- mined by applying Ohm's law to the voltage source:

$I = E/Rs$ .

the equivalence between a current source and a voltage source exists only at their external terminals.




EXAMPLE : For the circuit in Fig. below:




a. Determine the current IL.

b. Convert the voltage source to a current source.

c. Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).

solution:

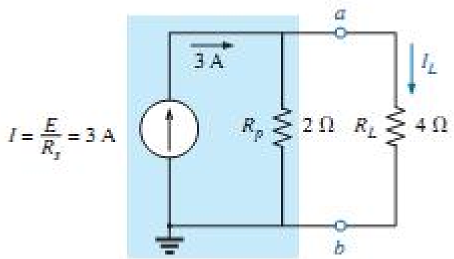
a. Applying Ohm's law:

$$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

b. Using Ohm's law again:

$$I = \frac{E}{R_s} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

and the equivalent source appears in Fig. with the load reapplied.



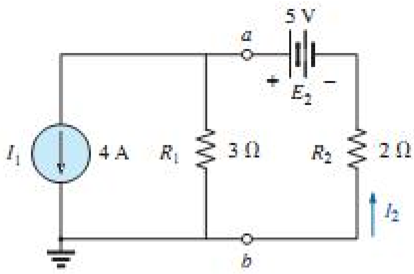

c. Using the current divider rule:

$$I_L = \frac{R_p I}{R_p + R_L} = \frac{(2\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \frac{1}{3}(3\text{ A}) = \mathbf{1\ A}$$

We find that the current $I_L$ is the same for the voltage source as it was for the equivalent current source—the sources are therefore equivalent.

Example:

Determine current $I_2$ for the network in Fig.

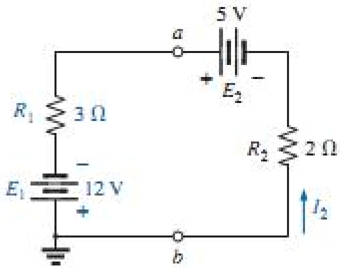


Solution:

For the source conversion:

$$E_1 = I_1 R_1 = (4\ \text{A})(3\ \Omega) = 12\ \text{V}$$

and $$I_2 = \frac{E_1 + E_2}{R_1 + R_2} = \frac{12\ \text{V} + 5\ \text{V}}{3\ \Omega + 2\ \Omega} = \frac{17\ \text{V}}{5\ \Omega} = \mathbf{3.4\ A}$$



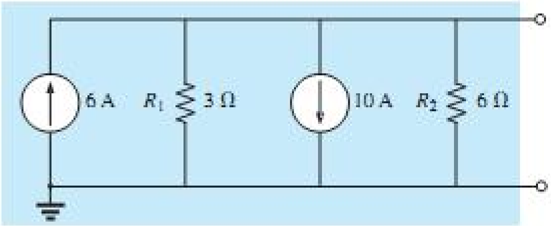

## CURRENT SOURCES IN PARALLEL:

We found that voltage sources of different terminal voltages cannot be placed in parallel because of a violation of Kirchhoff's voltage law.

Similarly,

current sources of different values cannot be placed in series due to a violation of Kirchhoff's current law. However, current sources can be placed in parallel just as voltage sources can be placed in series. In general, two or more current sources in parallel can be replaced by a single current source having a magnitude determined by the difference of the sum of the currents in one direction and the sum in the opposite direction. The new parallel internal resistance is the total resistance of the resulting parallel resistive elements.

EXAMPLE : Reduce the parallel current sources in Fig. to a single current source.



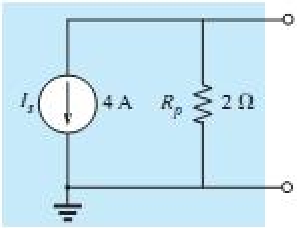

Solution:

$$I = 10\ \text{A} - 6\ \text{A} = \mathbf{4\ A}$$

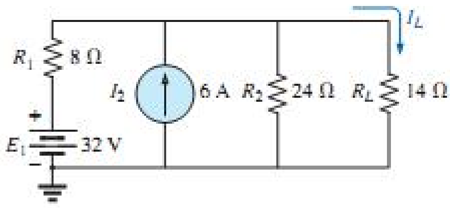
with the direction of the larger.

The net internal resistance is the parallel combination of resistors, $R_1$ and $R_2$:

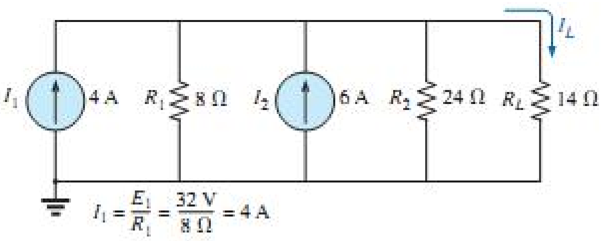
$$R_p = 3\ \Omega \parallel 6\ \Omega = \mathbf{2\ \Omega}$$




EXAMPLE : Reduce the network in Fig. below to a single current source, and calculate the current through $R_L$ .



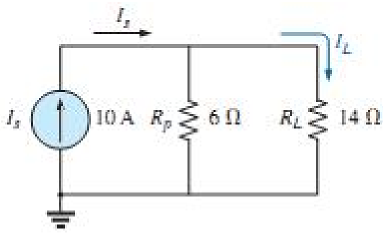

Solution:




$$I_s = I_1 + I_2 = 4\ \text{A} + 6\ \text{A} = \mathbf{10\ A}$$

and $$R_s = R_1 \parallel R_2 = 8\ \Omega \parallel 24\ \Omega = \mathbf{6\ \Omega}$$

Applying the current divider rule to the resulting network

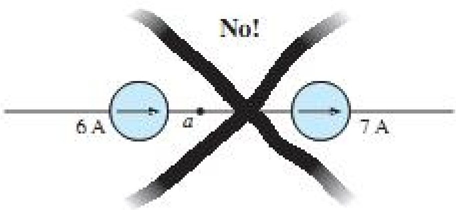



$$I_L = \frac{R_p I_s}{R_p + R_L} = \frac{(6\ \Omega)(10\ \text{A})}{6\ \Omega + 14\ \Omega} = \frac{60\ \text{A}}{20} = \mathbf{3\ A}$$

Note :

***current sources of different current ratings are not connected in series,***

just as voltage sources of different voltage ratings are not connected in parallel.



SUPERPOSITION THEOREM

The superposition theorem is unquestionably one of the most powerful in this field. It has such widespread application that people often apply it without recognizing that their exercises are valid only because of this theorem.

The superposition theorem states the following:

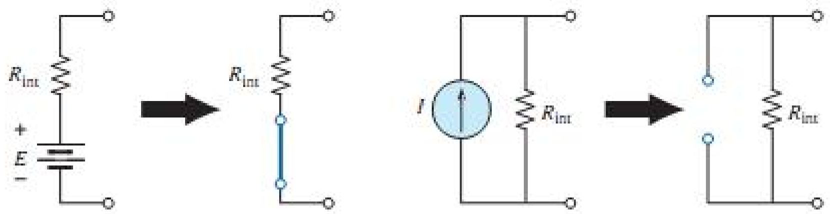
The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

-In other words, this theorem allows us to find a solution for a current or voltage using only one source at a time.

when removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network.

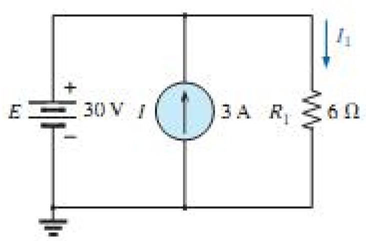
Setting a current source to zero amperes is like replacing it with an open circuit. Therefore, when removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.

The above statements are illustrated in Fig. below :



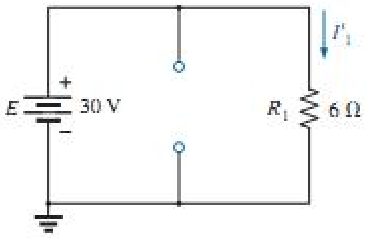

Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

EXAMPLE : Using the superposition theorem, determine current $I_1$ for the network in Fig.



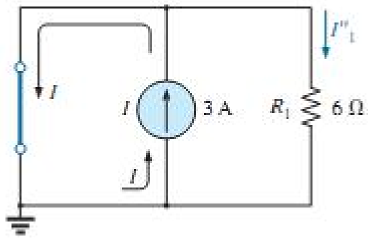
Solution:

First let us determine the effects of the voltage source by setting the current source to zero amperes as shown in Fig.




$$I'_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\text{ V}}{6\ \Omega} = 5\text{ A}$$

Now for the contribution due to the current source. Setting the voltage source to zero volts results in the network in Fig. Since the source current takes the path of least resistance, it chooses the zero ohm path of the inserted short-circuit equivalent, and the current through $R_1$ is zero amperes. This is clearly demonstrated by an application of the current divider rule as follows:

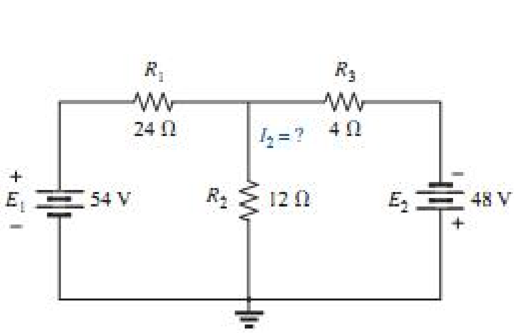



$$I''_1 = \frac{R_{sc}I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\text{ A}$$

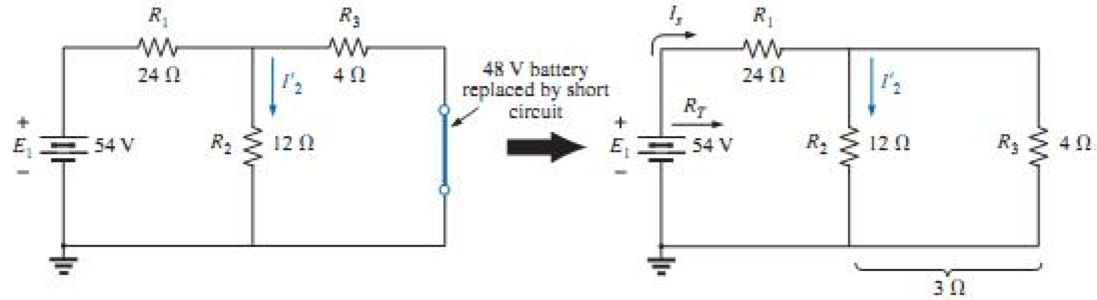
$$I_1 = I'_1 + I''_1 = 5\text{ A} + 0\text{ A} = \mathbf{5\ A}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\text{ V}}{6\ \Omega} = \mathbf{5\ A}$$

EXAMPLE : Using the superposition theorem, determine the current through the 12Ω resistor in Fig.



Solution:




The total resistance seen by the source is therefore

$$R_T = R_1 + R_2 \| R_3 = 24\ \Omega + 12\ \Omega \| 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

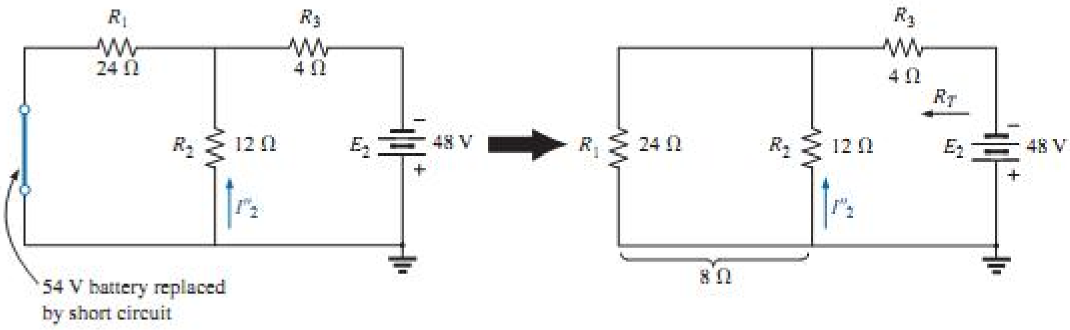
and the source current is

$$I_s = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A}$$

Using the current divider rule results in the contribution to $I_2$ due to the 54 V source:

$$I'_2 = \frac{R_3 I_s}{R_3 + R_2} = \frac{(4\ \Omega)(2\ \text{A})}{4\ \Omega + 12\ \Omega} = 0.5\ \text{A}$$

If we now replace the 54 V source by a short-circuit equivalent, the network in Fig. results. The result is a parallel connection for the 12Ω and 24Ω resistors.




Therefore, the total resistance seen by the 48 V source is

$$R_T = R_3 + R_2 \| R_1 = 4\ \Omega + 12\ \Omega \| 24\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

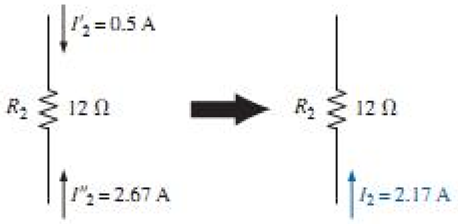
and the source current is

$$I_s = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A}$$

Applying the current divider rule results in

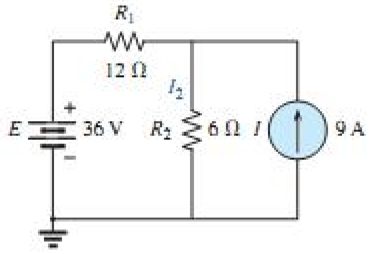
$$I''_2 = \frac{R_1(I_s)}{R_1 + R_2} = \frac{(24\ \Omega)(4\ \text{A})}{24\ \Omega + 12\ \Omega} = 2.67\ \text{A}$$

$$I_2 = I''_2 - I'_2 = 2.67\ \text{A} - 0.5\ \text{A} = \mathbf{2.17\ A}$$



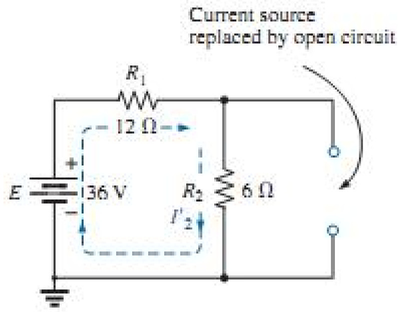

EXAMPLE :

a. Using the superposition theorem, determine the current through resistor R2 for the network in Fig.

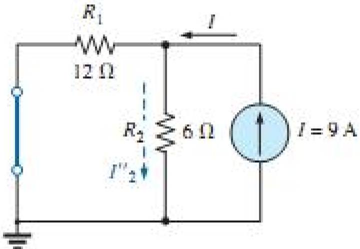



Solution :

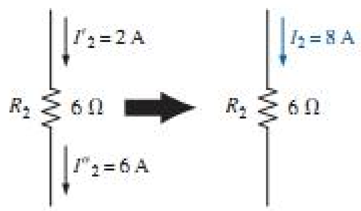
a)




$$I'_2 = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\text{ V}}{12\ \Omega + 6\ \Omega} = \frac{36\text{ V}}{18\ \Omega} = 2\text{ A}$$

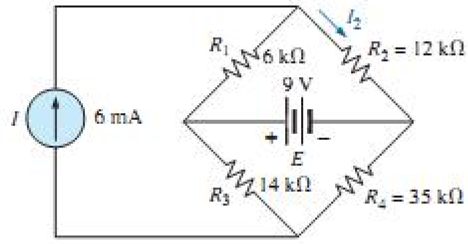



$$I''_2 = \frac{R_1(I)}{R_1 + R_2} = \frac{(12\ \Omega)(9\text{ A})}{12\ \Omega + 6\ \Omega} = 6\text{ A}$$

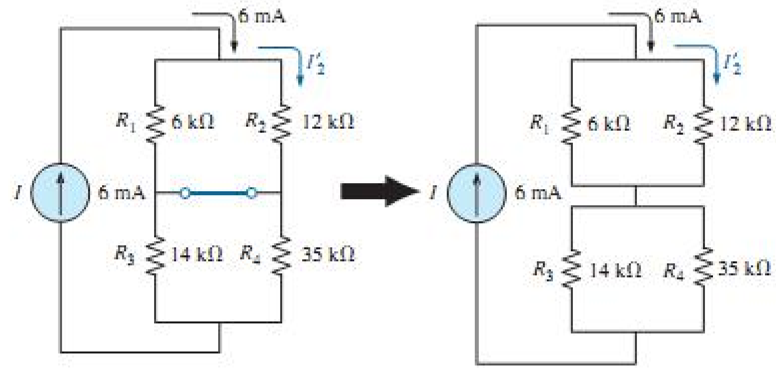



$$I_2 = I'_2 + I''_2 = 2\text{ A} + 6\text{ A} = \mathbf{8\ A}$$

EXAMPLE : Using the principle of superposition, find the current $I_2$ through the 12 kΩ resistor in Fig.




Solution: Considering the effect of the 6 mA current source




Current divider rule:

$$I'_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(6\text{ k}\Omega)(6\text{ mA})}{6\text{ k}\Omega + 12\text{ k}\Omega} = 2\text{ mA}$$

Considering the effect of the 9 V voltage source (Fig 9.17):

$$I''_2 = \frac{E}{R_1 + R_2} = \frac{9\text{ V}}{6\text{ k}\Omega + 12\text{ k}\Omega} = 0.5\text{ mA}$$

Since $I'_2$ and $I''_2$ have the same direction through $R_2$, the desired current is the sum of the two:

$$\begin{aligned} I_2 &= I'_2 + I''_2 \\ &= 2\text{ mA} + 0.5\text{ mA} \\ &= \mathbf{2.5\text{ mA}} \end{aligned}$$